# HOMMAGE À NICOLAS BERGERON

*par* Laurent CLOZEL, Tsachik GELANDER, Alan REID, Akshay VENKATESH, Daniel WISE, *et* Sébastien BOUCKSOM

En 2019, Nicolas Bergeron a pris la suite de Claire Voisin en tant que rédacteur-en-chef des Publications Mathématiques de l'IHES. Il s'y est investi avec l'énergie et l'enthousiasme communicatif que toutes celles et ceux qui ont eu la chance de croiser son chemin lui connaissaient—œuvrant notamment pour le passage en accès libre de ce journal—jusqu'à ce que la maladie l'empêche de poursuivre cette aventure.

Nicolas Bergeron était Professeur à Sorbonne Université et membre du Département de Mathématiques et Applications de l'École normale supérieure–Paris Sciences & Lettres. Sa recherche mathématique, couvrant un large spectre mêlant topologie, théorie des groupes et des espaces symétriques, théorie des nombres, et géométrie algébrique, a donné lieu à de nombreux résultats de tout premier plan, qui lui ont notamment valu d'être conférencier invité au congrès européen de mathématiques en 2016, au congrès international des mathématiciens en 2018, et de recevoir en 2023 le Prix Fondé par l'État de l'Académie des Sciences.

Nicolas Bergeron était également très impliqué dans l'enseignement et la diffusion des mathématiques. Passionné par les travaux de Henri Poincaré, il a en particulier travaillé sur le projet Analysis Situs et le livre *Uniformisation des surfaces de Riemann* avec le collectif Henri Paul de Saint-Gervais [SG10].

Le comité éditorial de ce journal, profondément attristé par l'annonce récente de la disparition de Nicolas, a souhaité lui dédier le présent numéro, et recueillir dans ce qui suit quelques textes de certains de ses proches collaborateurs, évoquant sa mémoire et ses mathématiques.


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**Laurent Clozel**
***Travaux de Bergeron sur la cohomologie des espaces localement symétriques***

1. La thèse de Bergeron [Ber00], soutenue en l'an 2000, portait essentiellement sur les propriétés homologiques des cycles (de dimension arbitraire) totalement géodésiques dans les variétés hyperboliques compactes. En particulier, Bergeron démontrait dans de nombreux cas la non–nullité de $H^i(\tilde{S}, \mathbf{C})$, $i \leq \dim S$ pour un revêtement fini $\tilde{S}$ d'une telle variété S (le cas $i = 1$ étant une conjecture de Thurston). Pour cela, Bergeron dégageait l'hypothèse fondamentale suivante (ici reformulée d'après [Ber03]). On considère des variétés hyperboliques **réelles** ou **complexes**. Soit G un groupe défini sur $\mathbf{Q}$ et tel que $G(\mathbf{R}) \cong SO(n, 1)$ ou $G(\mathbf{R}) \cong SU(n, 1)$. Soit $X_G$ l'espace symétrique de $G(\mathbf{R})$, et $d_G$ la dimension **réelle** de $X_G$.

Soit $\lambda_1^i$ la plus petite valeur propre non nulle du laplacien de Hodge sur les $i$–formes sur $\Gamma\backslash X_G$, $\Gamma \subset G(\mathbf{Q})$ étant un sous–groupe de congruence.

*Conjecture A*. — *[Ber03] Pour tout* $i \leq \frac{d_G}{2} - 1$, *il existe* $\varepsilon(G, i) > 0$ *tel que, pour* ***tout*** *sous–groupe de congruence* $\Gamma \subset G(\mathbf{Q})$, $\lambda_1^i(\Gamma\backslash X_G) \geq \varepsilon(G, i)$.

Nous n'allons formuler la conjecture géométrique associée que pour la cohomologie [Ber03] (on renvoie aux articles de l'auteur pour les conjectures homologiques). Soit $H \subset G$ un $\mathbf{Q}$–sous–groupe tel que $H(\mathbf{R}) = SO(k, 1)$ (resp. $SU(k, 1)$). On considère $Sh^0 G := \varprojlim_{\Gamma} (\Gamma\backslash X_G)$ et

$$H^\bullet\left(Sh^0 G\right) := \varinjlim_{\Gamma} H^\bullet(\Gamma\backslash X_G),$$

la limite étant prise sur les sous–groupes de congruence ; de même $H^\bullet(Sh^0 H)$. Soit $j : Sh^0 H \to Sh^0 G$ l'application naturelle, et, pour $g \in G(\mathbf{Q})$, $j_g$ l'application $j$ translatée par $g$.

*Conjecture B*. — *Pour* $i \leq \frac{1}{2} d_H$, *l'application*

$$H^i\left(Sh^0 G\right) \to \prod_{g \in G(\mathbf{Q})} H^i\left(Sh^0 H\right)$$

*donnée par* $\prod_g j_g^*$ *est injective.*

La conjecture B est donc une propriété d'injectivité « virtuelle » : on passe à la limite sur les revêtements donnés par les groupes de congruence.

2. Par restriction à la diagonale, la Conjecture B implique la non–nullité « virtuelle » du cup–produit dans certains degrés. Pour les variétés hyperboliques complexes,

elle fut démontrée par Venkataramana après des travaux d'Harris et Li. Dans ce cas, on peut la considérer comme un analogue du théorème de Lefschetz sur les sections hyperplanes.

3. On introduit ensuite la description de la cohomologie de $\Gamma\backslash X_G$ par la formule de Matsushima. Alors $H^\bullet(\Gamma\backslash X_G)$ est déterminé par l'occurence dans $L^2(\Gamma\backslash G)$ de certaines représentations unitaires, notées $A(\mathfrak{q})$ par Vogan–Zuckerman, et dont la cohomologie continue $H^\bullet_{ct}(G, A(\mathfrak{q}))$ est non–nulle. L'analyse du complexe de Matsushima montre alors que la conjecture $A(i)$, i.e. A pour $i$ fixé, est vérifiée si toute représentation $A(\mathfrak{q})$ telle que $H^i_{ct}(G, A(\mathfrak{q})) \neq 0$ est **isolée** dans le dual unitaire $\hat{G}$ de G. Un théorème de Vogan décrit explicitement ces représentations isolées ; il est redémontré en détail dans [Ber03, §4]. Bergeron prouve que si $A(i)$ $(i \leq \frac{1}{2}d_H - 1)$ est vraie pour H, alors $B(i+1)$ est vraie pour la paire $(G, H)$ [Ber03, Thm 2.4] (pour les groupes $SO(n, 1)$ et $SU(n, 1)$, les représentations cohomologiques ne sont pas isolées).

On peut aussi démontrer des variantes de $B(i)$ dans le cas où une représentation $\pi = A(\mathfrak{q})$ de G admet une restriction discrète à H, contenant une représentation $\rho = A(\mathfrak{q}_H)$ (notation évidente). Voir [Ber06, §5].

4. A l'aide de toutes ces méthodes – à commencer par l'analyse difficile, inspirée des travaux de Kudla et Millson, permettant de construire par « sommation de Poincaré » la classe de cohomologie duale à un cycle [Ber00] – Bergeron démontre toute une variété de « théorèmes de Lefschetz » dans le cadre automorphe. On renvoie à [Ber03], ainsi qu'au mémoire exhaustif [Ber06]. Noter que ces résultats ne se limitent pas au cas des variétés hyperboliques !

5. Bergeron obtient une application spectaculaire de ces méthodes au cas des cup–produits [Ber04]. Si $G = U(p, q)$, les représentations $A(\mathfrak{q})$ peuvent être paramétrées par des couples de partitions $(\lambda, \mu)$. Notons $A(\mathfrak{q}) = A(\lambda, \mu)$. Soit de même $A(\alpha, \beta)$ une autre représentation cohomologique de G.

Bergeron donne alors un critère purement combinatoire sur les deux couples de partitions impliquant que le cup–produit de classes primitives provenant de $A(\lambda, \mu)$ et $A(\alpha, \beta)$ est virtuellement non nul. C'est en particulier le cas en « petits » degrés.

6. Revenons sur la Conjecture A. Pour les groupes de rang (réel) 1, elle ne découle pas d'une propriété du dual unitaire « abstrait » $\hat{G}$. Mais Arthur a proposé une description du dual **automorphe** $\hat{G}_{Aut}$ (représentations apparaissant dans $L^2(\Gamma\backslash G)$.) En 2013, Arthur a démontré ses conjectures pour les groupes classiques quasi–déployés. Comme annoncé dans [BC05], nous pouvions alors démontrer la Conjecture A pour les groupes de type $SO(n, 1)$ [BC13] ou $SU(n, 1)$. Les bornes $\varepsilon(G, i)$ sont explicites et démontrent[1] une conjecture de Burger et Sarnak. Les conséquences topologiques s'ensuivent.[2]

7. Au terme de ce périple nous pouvons revenir sur la conjecture de Thurston. Il existe des variétés hyperboliques compactes de dimension 7 associées à des $\mathbf{Q}$–formes

[1] A $\varepsilon$ près, cf. [BC13].
[2] Les démonstrations des appendices du livre d'Arthur n'ont pas été publiées. Ces résultats sont donc conditionnels.

« exotiques » de $G = SO(7, 1)$ (trialité). Si $S = \Gamma\backslash X_G$ est une telle variété, $\Gamma$ étant un sous-groupe de congruence dans $G(\mathbf{Q})$, tout revêtement $S' = \Gamma'\backslash X_G$ de $S$ ($\Gamma'$ de congruence) a un premier nombre de Betti égal à 0, cf. [BC17]. La conjecture de Thurston ne peut être vraie que s'il existe des sous-groupes arithmétiques non de congruence.[3]

## Tsachik Gelander
### *Moments with Nicolas*

We were sitting in Nicolas' office shortly after Francois Labourie introduced us, discussing random mathematical ideas and taking notes on the board. It was at the winter of 2001. When the board was almost full, Nicolas added at the bottom "and this implies GRH", in case someone walks in and wonders what we are doing. I was a visiting student from the Hebrew University and he was a young faculty at Paris Sud. I was about to give a talk at the Topology and Dynamics seminar at Orsay, my first seminar talk ever outside my home institute at Jerusalem. When I got up to leave and prepare for my talk he said "I suppose they told you about the seminar tradition". What tradition ? "That the speaker should start with a joke. But it has to be authentic, something original or a personal story". Instead of preparing for the talk I spent the next couple of hours trying to come up with a joke. I started my lecture by saying that I'm going to give an elementary proof for the four color theorem. The "proof" started with a reduction that any map can be drawn on a cow and ended with the fact that every cow can be colored by two colors, so we actually obtained an even stronger result. There were many famous mathematicians in the crowd, all looking at me with shocked faces, except Nicolas, who was sitting at the back completely amused and satisfied by the situation. A year later when I came to speak in the same seminar, he told me that they changed the tradition and now the speaker should tell about his first sexual experience. This time I didn't fall into the trap.

Our first collaboration was a modest one. To prove the obvious fact that strong (Mostow) rigidity implies weak (local) rigidity [BG04]. Well it's not completely obvious since it is false for non-compact manifold of dimension 3. So we had to show that all these 3d deformations (such as Dehn twists) cannot have analogs in other dimensions. After that we had several more ambitious projects concerning the geometry of higher rank manifolds as well as higher dimensional hyperbolic manifolds. We spent many hours working in Luxembourg gardens. As we enjoyed our work, we continued these projects even after understanding that we would probably never complete or publish it. The notes taken in his and my handwriting are still collecting dust in one of my drawers.

When working on a mathematical problem together he was usually the optimist while I was the pessimist. Surprisingly, his optimism often justified itself and his wild conjectures turned out to be correct. I suppose this indicates his ability to understand

[3] La remarque de la note 2 s'applique ici aussi.

and see beyond the details. I learned a lot from Nicolas, not just mathematics but also about myself and other things in life. He would see clearly things that I missed, that only became obvious once he said them.

I would like to say few words about the 7 authors work (sometimes referred as the 7 samurai paper). The project was suggested by Miklos Abert and together we chose who to invite. Sometime in 2011, we all met for a week and "locked" ourselves in a classroom in Paris 6, taking turns on the board and suggesting ideas. I was quite pessimistic at the beginning but by the end of the week we already had a solid scheme. A few months later we met for another week, this time in Budapest, by the end of which we already had a theorem and an announcement [A+11]. It took a couple of more years to complete the paper [A+12, A+17, A+20]. We developed from scratch the theory of invariant random subgroups of Lie groups and applied it to study asymptotic invariants of arithmetic groups. For example we obtained a uniform estimate on normalized Betti numbers improving many earlier works [A+11, ABBG23] but we also obtained uniform estimates on much more general analytic invariants. This work contained so much new mathematics that each of us could add his contribution, but Nicolas' contribution was especially essential. There are big parts of this work that none of us could do except for Nicolas.

I met him mostly in Paris since he didn't like to travel a lot. Every time I came he would take me to a movie, and always a thoughtfully chosen one. In fact, most of my visits to the cinema over the last 23 years were with Nicolas. The last time he took me to the cinema was last year. He had electrodes attached to his head covered with a cowboy hat and carried the generator in his bag. I remember the overwhelming feeling I had when the lady sitting behind us asked him to remove his hat. He simply apologized and took his hat off.

**Alan Reid**
***Nicolas Bergeron***

It was with a profound sense of shock and sadness that I learned of the passing of Nicolas Bergeron in mid February of this year. I had been aware of his health issues, going back to Spring 2022 when I spent the semester at Max-Planck-Institüt, Bonn, and co-organized a workshop there "*Arithmetic Groups and 3-Manifolds*" to which Nicolas was invited to speak. Regrettably because of his health issues, he could not. More recently, I had email correspondence with him, and in fact, as recently as November 2023, we had an email exchange in which we tentatively arranged to meet in Paris in May when I would visit IHP for a week. Indeed, I am writing this article a few days before my departure to Paris, and feel his loss keenly once again.

I had already heard of Nicolas whilst he was a graduate student of Jean-Pierre Otal at ENS Lyon, and (I think) we met for the first time at a conference "*Aspects of Hyperbolic Geometry*" in Fribourg, Switzerland in October 2001. Nicolas broad interests in the geometry and topology of locally symmetric spaces, their fundamental groups, and connections to number theory and automorphic forms are very close to many of my own interests, and over the years I have followed his work closely.

There are simply too many highlights of Nicolas work to discuss in detail or even summarize in this short note, but the reader would be well served to read Nicolas paper "*Hodge theory and cycle theory of locally symmetric spaces*", which appeared in the Proceedings of the 2018 international Congress of Mathematicians, held in Rio de Janeiro [Ber18]. This gives a clear and coherent account of some of Nicolas (and collaborators) excellent work connecting the topology of locally symmetric spaces, particularly their cohomology, the structure of their fundamental groups, and connections to number theory and automorphic forms, most notably through the theory of automorphic representations. In particular, this paper provides an elegant survey of Nicolas (and co-authors) beautiful work on the cohomology of locally symmetric spaces and to explain the parallels with classical Hodge theory. As the abstract to this paper succinctly states :

"*The unity behind these results is motivated by a vague but fruitful analogy between locally symmetric spaces and projective varieties.*"

The 2018 ICM in Rio was the last time I was able to spend an extended period of time chatting to Nicolas about mathematics and generally hanging out. I look back on that time fondly : he is, and will continue to be, sorely missed both for his mathematics and his warm and engaging personality.

**Akshay Venkatesh**
***Memories of my friend Nicolas Bergeron***

I started working with Nicolas Bergeron in late 2007, when he visited the Courant Institute. We had crossed paths several times already; we had a common mentor in Laurent Clozel, who told me that I should meet this unique geometer who had managed to learn the language of automorphic forms. We became friends immediately, sharing interests both within and without mathematics (Nicolas would not forgive me if I do not mention that many a working session in our future would be carried out to the soundtrack of a Bob Dylan album). Despite many common interests, our backgrounds were quite different; Nicolas was trained as a geometer, and I as an analytic number theorist. This distinction was a key source of creative tension in our collaboration.

At the time that Nicolas and I started working together, Frank Calegari and I had been examining numerical computations by Nathan Dunfield about the homology of Bianchi groups. (An example of a Bianchi group is

$$\Gamma = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbf{Z}[\sqrt{-1}] : ad - bc = 1 \right\}$$

of unimodular $2 \times 2$ matrices with Gaussian integer coefficients ; more generally we might replace $-1$ by another negative integer, or impose congruence constraints.) Dunfield's computations suggested a number of striking phenomena ; among them was that the abelianization $\Gamma^{ab}$ of such a group, or equivalently its first homology $H_1(\Gamma, \mathbf{Z})$, could contain an enormous amount of *torsion*.

Bianchi groups are specific examples of *arithmetic groups* – informally, classical matrix groups with integral entries. The homology of such groups with complex coefficients has received an enormous amount of attention, because of its close relation to the theory of automorphic forms. But at the time of our work, there had been relatively little systematic study of the possibility of torsion in this homology, and the idea that it might be abundant had, apparently, not been seriously examined.

I discussed all this with Nicolas, who in his characteristic way immediately took an interest and invited me to Paris to continue discussing it. (I still remember the extraordinary patience with which he navigated an apparent infinity of administration required to arrange this visit.) In the course of a month we were able to establish the main result of our first paper [BV13] : we produced (among other things) many examples of sequences $\Gamma \supset \Gamma_1 \supset \Gamma_2 \supset \ldots$ of arithmetic groups and suitable $\Gamma$-modules M for which the amount of torsion homology of $\Gamma_N$ with coefficients in M grows *exponentially* with the index $[\Gamma : \Gamma_N]$. This result seemed out of reach to me before I arrived in Paris ; we were carried over the difficulties only by Nicolas's relentless (reckless ?) optimism. We continued to examine the phenomenon in our later work [BSV16], joined by Haluk Sengun, and there remains much to understand here. This collaboration had a great influence on my research direction ; trying to understand better the limits of our method led me, later, to a sequence of papers relating motivic cohomology and the cohomology of arithmetic groups.

Nicolas visited me again, in San Francisco around 2016, and we started on another project, in which we were soon joined by Luis Garcia and Pierre Charollois. But I became busy with other things, and eventually bowed out. My loss. For they went on to uncover numerical evidence for an amazing new phenomenon : the elliptic $\Gamma$ function (a rather youthful special function, which looks a bit like a $\theta$-function but is defined by a double product) has entirely unexpected algebraicity properties when the inputs are chosen to be certain cubic irrationalities. The structure possessed by these evaluations suggests that much more is going on behind the scenes. This is all laid out in a paper [BCG23] that appeared just months before Nicolas's death ; it marks, I think, the sighting of a new and unexplored land for number theory.

My most vivid memory of Nicolas is of laughing together – about the absurdity of our latest failed proof, about our paper being rejected, about my incessant complaints regarding the font of *Publications Math IHÉS*, and even about his illness. This joyful way of living was reflected also in Nicolas's approach to research, which always had a kind of marvellous lightness. This lightness was not opposed to depth, but something else ; a delight, a playful aspect. Thinking about this I feel his influence on me – for how else, indeed, should we do mathematics ?

**Daniel Wise**
***Nicolas Bergeron***

I first heard of Nicolas from our mutual friend Frédéric Haglund who told me of the tremendously talented Nicolas, and of his interest and engagement with work that Frédéric and I had done a few years earlier. Our own results on special cube complexes included a curious criterion for virtual specialness - the double coset separability criterion : If X is a compact nonpositively curved cube complex, then X has a finite special cover if and only if the double coset $\pi_1 U \pi_1 V$ is separable in $\pi_1 X$ whenever $U, V$ are immersed hyperplanes passing through the same point.

Nicolas had already, I think in his PhD thesis, engaged with such hyperplanes but within hyperbolic manifolds, and he had already understood their double coset separability. Upon learning of this connection, Nicolas wrote a text on simple-type arithmetic hyperbolic manifolds that combined Bergeron with Haglund-Wise. Quite a nice paper that he wrote and, as Frédéric said, "put our names on".

Shortly thereafter, winter 2006/2007, Nicolas came to visit me in Montreal. He showed up exhausted from a long flight, and immediately joined Yael and our family for Shabbat dinner. He brought us delicious chocolates which we enjoyed together for dessert and it was a pleasure to have him as our guest for the week.

There were many mathematical forays during his visit, but the most interesting was the "boundary criterion for cubulation", which asserts that for a word-hyperbolic group G, there exists a proper cocompact action of G on a CAT(0) cube complex provided that for any points $p, q \in \partial G$, there exists a quasiconvex subgroup $H \subset G$ with $\partial H$ separating $p, q$. It didn't seem to be a priority to articulate what we had done, as the main application we knew was to give a softer explanation of cubulation for the simple-type hyperbolic arithmetic lattices. And that was already done with a straightforward and explicit geometric argument.

We had an unpleasant experience when we found that my car was out of gas when we returned from working hard in a cafe. Despite the freezing cold, Nicolas was a good sport, and very helpful in resolving the problem, and together we hid what happened from Yael, who was judgmental of my petrol disability (to be fair, the gas-gauge was broken). Celine joined Nicolas in Montreal near the end of our research visit, and the four of us had a marvelous night out, temporarily free parents of small kids comparing life.

At some point during his visit, we wanted to compare ideas we had about measured wall spaces to ideas in a recent text of Delzant-Gromov on "quasiconvex cuts". I glanced at the first paragraph and bet a nickle that the authors would *never* define "quasiconvex-cut". He happily accepted the bet and we skimmed through the paper together. Myself, I was just scanning for definitions to be sure I'd win. Upon reaching the end, I found to my amazement that Nicolas already had a remarkably good grasp of the actual mathematics in the text, and though amused by the lack of a formal definition, he was perfectly content to catch on. Math was profoundly easy for him, and he was a powerful reader. I was careful not to collect the nickle so that he would permanently owe me.

In 2009, after hearing of the Kahn-Markovic result, we wrote down our earlier boundary cubulation criterion which now gave cubulation as a corollary of the ubiquity of quasifuchsian surfaces in closed hyperbolic 3-manifolds given by Kahn-Markovic. I did a sloppy job writing an attempted early version and Nicolas turned it into actual math over a few emails exchanged while I did a pitstop in Paris to visit Frédéric on my way back from sabbatical in Jerusalem, and Nicolas was on a summer vacation with his family. When I finally returned to Montreal, I had suggested to Nicolas the way I hoped it would generalize to the relatively hyperbolic setting, and he not only determined how to articulate a proof, but quickly learned and then taught me Bowditch's approach to relative hyperbolicity. Maybe it was the first time I started to understand that business - it was remarkably easy for Nicolas to expand his range, to move between different fields and acquire expertise.

I had many occasions over the years, at conferences and visits between Paris and Montreal, to spend time with Nicolas. Our energetic coffee/lunch meetings moved back and forth between exchanges about math and about life. Always possibilities and promise, always laughter, and a fun mixture of cynicism and optimism. Always a glimmer in his eyes. I feel like we saw eye-to-eye even when we disagreed and we enjoyed our heated discussions. There was one issue where we had disagreed one way, and then both reversed our positions and disagreed again when we revisited the point some years later! Though totally fluent in English, Nicolas occasionally complained that he was "more convincing in French than in English", but that just seemed a rhetorical tactic to me, as he was triumphantly expressive and communicative in English.

I am grateful to have had this friend, and I fully realize that I am just one of many among Nicolas' broad network of deep connections. We will all miss him.

**Declarations :**

**Competing Interests**

The authors declare no competing interests.

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## Publisher's Note

L. C.
Université Paris-Saclay,
Orsay, France

T. G.
Northwestern University,
Evanston, IL, USA

A. R.
Rice University,
Houston, TX, USA

A. V.
Princeton Institute of Advanced Study,
Princeton, NJ, USA

D. W.
McGill University,
Montreal, QC, Canada

S. B.
CNRS,
Institut de Mathématiques de Jussieu-Paris Rive Gauche,
Paris, France
sebastien.boucksom@gmail.com

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